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Depth and structural index from normalized local wavenumber of 2D magnetic anomalies

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ABSTRACT

Recent improvements in the local wavenumber approach have made it possible to estimate both the depth and model type of buried bodies from magnetic data. However, these improvements require calculation of third-order derivatives of the magnetic field, which greatly enhances noise. As a result, the improvements are restricted to data of high quality. We present an alternative method to estimate both the depth and model type using the first-order local wavenumber approach without the need for third-order derivatives of the field. Our method is based on normalization of the first-order local wavenumber anomalies and provides a generalized equation to estimate the depth of some 2D magnetic sources regardless of the source structure. Information about the nature of the sources is obtained after the source location has been estimated. The method was tested using synthetic magnetic anomaly data with random noise and using three field examples.

INTRODUCTION

Recently, Thurston and Smith (1997) presented the SPI method as a new approach for interpretation of magnetic data. This method requires second-order derivatives of the total field and uses a term known as the first-order local wavenumber to estimate the depth of buried magnetic bodies. The first-order local wavenumber is defined as the rate of change of the local phase, which is one of three attributes derived from the complex analytic signal (Nabighian 1972; Thurston and Smith 1997). One of the main characteristics of the local wavenumber approach for the interpretation of magnetic data is that it has a simple bell-like shape (similar to the shape of the analytic signal) over certain simple 2D magnetic bodies such as contacts, thin dikes, and horizontal cylinders. If the model type is assumed, the local wavenumber can provide information about the depth using simple calculations. However, the accuracy of the results depends upon how closely the assumed model approximates the real structure. Also, *a priori* knowledge of the nature of the source is not usually available. To enable estimation of both the location and nature of the source, Smith *et al.* (1998) and Thurston, Smith and Guillon (2002) introduced a second-order local wavenumber. However, the improved methods require calculation of third-order derivatives of the field, which greatly enhance noise. Therefore, these methods require data of high quality, or careful filtering.

We present a simple method to estimate the location and model type of 2D magnetic sources using normalization of the first-order local wavenumber. Once the horizontal location has been identified from the location of the peak of the wavenumber anomaly, the depth can be estimated without any prior information about the nature of the source. Then using the information about the horizontal location and depth, the nature of the source is also obtained.

NORMALIZED LOCAL WAVENUMBER

The first-order local wavenumber is given by Thurston and Smith (1997) as

$$k_1 = \frac{1}{|A|^2}\left(\frac{\partial^2 M}{\partial x \partial z}\frac{\partial M}{\partial x} - \frac{\partial^2 M}{\partial x^2}\frac{\partial M}{\partial z}\right), \tag{1}$$

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where $\partial M/\partial x$ and $\partial M/\partial z$ are the derivatives of the field M in the x- and z-directions, and $|A|$ is the amplitude of the analytic signal, expressed by Nabighian (1972) as

$$|A| = \sqrt{\left(\frac{\partial M}{\partial x}\right)^2 + \left(\frac{\partial M}{\partial z}\right)^2}. \tag{2}$$

The first-order local wavenumber magnetic anomalies over 2D contacts with an infinite depth extent, thin dikes with an infinite depth extent, and horizontal cylinders located at a horizontal location ($x = 0$) and at a depth h are given by Smith *et al.* (1998) as

$$k_1(x) = \frac{h}{x^2 + h^2} \text{ for a contact,} \tag{3a}$$

$$k_1(x) = \frac{2h}{x^2 + h^2} \text{ for a thin dike,} \tag{3b}$$

$$k_1(x) = \frac{3h}{x^2 + h^2} \text{ for a horizontal cylinder.} \tag{3c}$$

Equations (3a), (3b) and (3c) can be generalized as

$$k_1(x) = \frac{(\eta + 1)\, h}{x^2 + h^2}, \tag{4}$$

where η is a value characterizing the source geometry ($\eta = 0$ for a contact, 1 for a dike, and 2 for a horizontal cylinder) and is known as the structural index in the Euler method (Thompson 1982). Equation (4) shows that, for the above models, the local wavenumber has a maximum value, located directly above the source, which can be expressed as

$$k_1(0) = \frac{\eta + 1}{h}. \tag{5}$$

Normalizing the local wavenumber using the maximum value $k_1(0)$, we obtain

$$k_{1n}(x) = \frac{k_1(x)}{k_1(0)} = \frac{h^2}{x^2 + h^2}. \tag{6}$$

Equation (6) can be solved using non-linear inversion (Press *et al.* 1992) to estimate the depth. Once the depth is obtained, the structural index can be calculated directly from (5). Also, from the same data it is possible to estimate the structural index from (4) in a least-squares sense, using

$$\eta = \frac{\sum_{i=1}^{N} k_1(x_i)\, h_c/\left(x_i^2 + h_c^2\right)}{\sum_{i=1}^{N} \left[h_c/(x_i^2 + h_c^2\right]^2} - 1, \tag{7}$$

where N is the number of selected data and h_c is the estimated depth.

IMPLEMENTATION

Implementation of the present method is simple. Horizontal positions of the sources are first detected at the locations of the local wavenumber peaks. If the local wavenumber data are noisy, peaks of the analytic signal can be used. Generally, analytic signal anomalies are less affected by noise because they are calculated using only first-order derivatives of the field. Once the horizontal positions are detected, N data points around a peak are used to estimate the depth and structural index using (6) and (7), respectively. The choice of the number of data points to use is a function of the data quality and the degree of interference of anomalies from nearby sources. For isolated anomalies, larger window sizes (in terms of number of data and as a result, window length) are required to reduce the effect of noise. For multiple sources, smaller numbers of data points are appropriate to reduce interference effects of nearby sources.

THEORETICAL EXAMPLE

To demonstrate the feasibility of the present method, it has been tested using theoretical anomaly data for a 2D magnetic body contaminated with random noise. We also show side-by-side results from the iSPI (Smith *et al.* 1998) and multimodel (Thurston *et al.* 2002) methods. These methods use the second-order local wavenumber to obtain information about the depth and structural index. They estimate the depth using the equation

$$k_2(x) - k_1(x) = \frac{h}{x^2 + h^2}, \tag{8}$$

where k_2 is a second-order local wavenumber. In the iSPI method, the depth is estimated directly from the reciprocal of the local wavenumber at the maxima. The multimodel method uses non-linear inversion to estimate both the depth and horizontal location.

Total-field anomaly data were calculated along a 40 km profile striking S–N at intervals of 1 km over a thin vertical dike model trending E–W and located at the centre of the profile with a depth to its top of 6 km. The dike model has a magnetization of 10 A/m and considers only induction with an inclination of 60° and a declination of 0°. We contaminated the anomaly data by adding sets of background random noise with zero mean and different standard deviations ranging from 0.1 to 1 nT with an interval of 0.1 nT. For each noise level, first- and second-order local wavenumber data were computed using a fast Fourier transform (FFT).

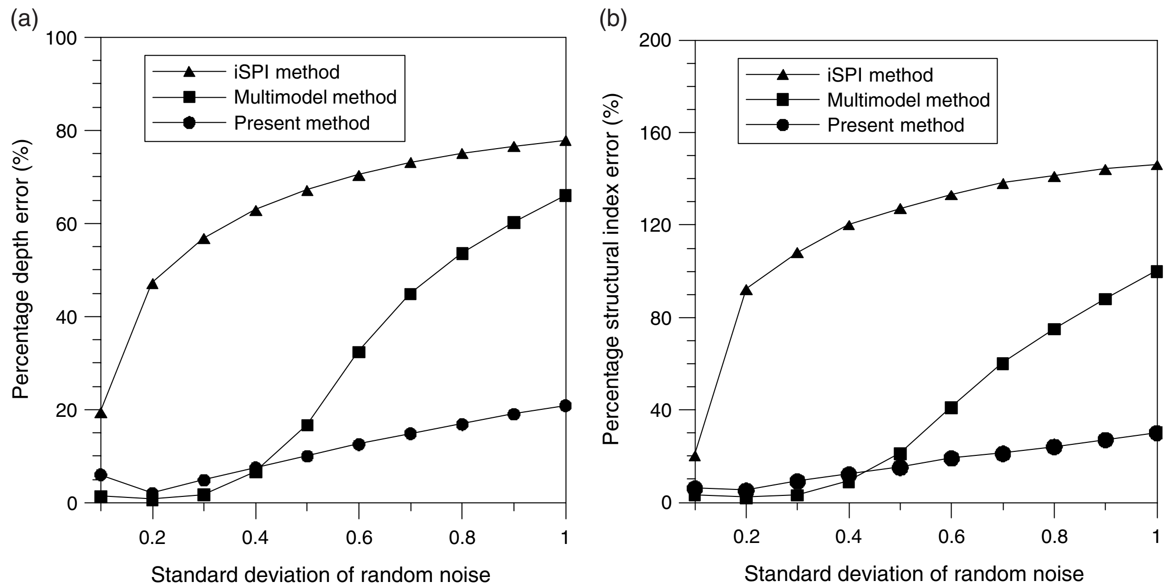

Figure 1 Comparison of the present method with the iSPI method (Smith *et al.* 1998) and the multimodel method (Thurston *et al.* 2002) using data from a theoretical anomaly of a thin dike model contaminated with different sets of random noise. (a) Percentage depth errors as a function of standard deviation of random noise; (b) percentage structural index errors as a function of standard deviation of random noise.

In testing the normalized local wavenumber method and the multimodel method, we used a data window of 21 data points. Figures 1(a) and 1(b) show the percentage depth and structural index errors obtained using the three methods. The presence of noise affects results of the normalized local wavenumber method as well as those of the iSPI and multimodel methods. Results of the multimodel method are better than those of the iSPI method. For low noise levels (less than 0.4 nT), the multimodel method and the normalized local wavenumber method give comparable results. This is expected since the multimodel method also utilizes a least-squares approach like the present method. However, for noise levels greater than 0.4 nT, results of the normalized local wavenumber method are significantly better for both the depth and structural index. An advantage of the multimodel method over the normalized local wavenumber method is that it is able to identify sloping, finite step, and thick dike models.

FIELD EXAMPLES

In this section, we tested the validity of the normalized local wavenumber method using two field examples over sources investigated by drilling. In a third example, we demonstrate the practical utility of the method to characterize anomalies due to unknown sources.

Examples over anomalies of known sources

Figure 2(a) shows a magnetic anomaly from the Pima copper mine, Arizona, USA (Gay 1963, fig. 10). The profile, 750 m in length, was digitized at intervals of 25 m. Drilling information showed that the mineralized zone is due to a thin dike body located at a depth of 64 m. Figure 2(b) shows the calculated first-order local wavenumber. Estimation of the horizontal location based on the local wavenumber peak was difficult, due to noise. The horizontal position of the source was located at a distance of 350 m using the peak of the analytic signal anomaly (Fig. 2b). The local wavenumber value at the estimated horizontal location was used to normalize the local wavenumber data. In the inversion, different data points centred above the estimated horizontal location (from 11 to 29 data points at intervals of two data points) were used. Estimates of the depth and structural index are shown in Figs 2(c) and 2(d), respectively. The estimated values of the structural index (0.73–0.93) indicate that the source can be approximated by a thin dike. The estimated depths (62–71 m) agree very well with the actual depth (64 m).

Figure 3(a) shows part of an aeromagnetic flight line over a magnetic body in the Matheson area of northern Ontario, Canada. The magnetic sensor was located in a bird about 70 m above the ground and the station spacing was approximately 12 m (Ontario Geological Survey 2000). The survey was flown with a flight line direction of N30°W, which is

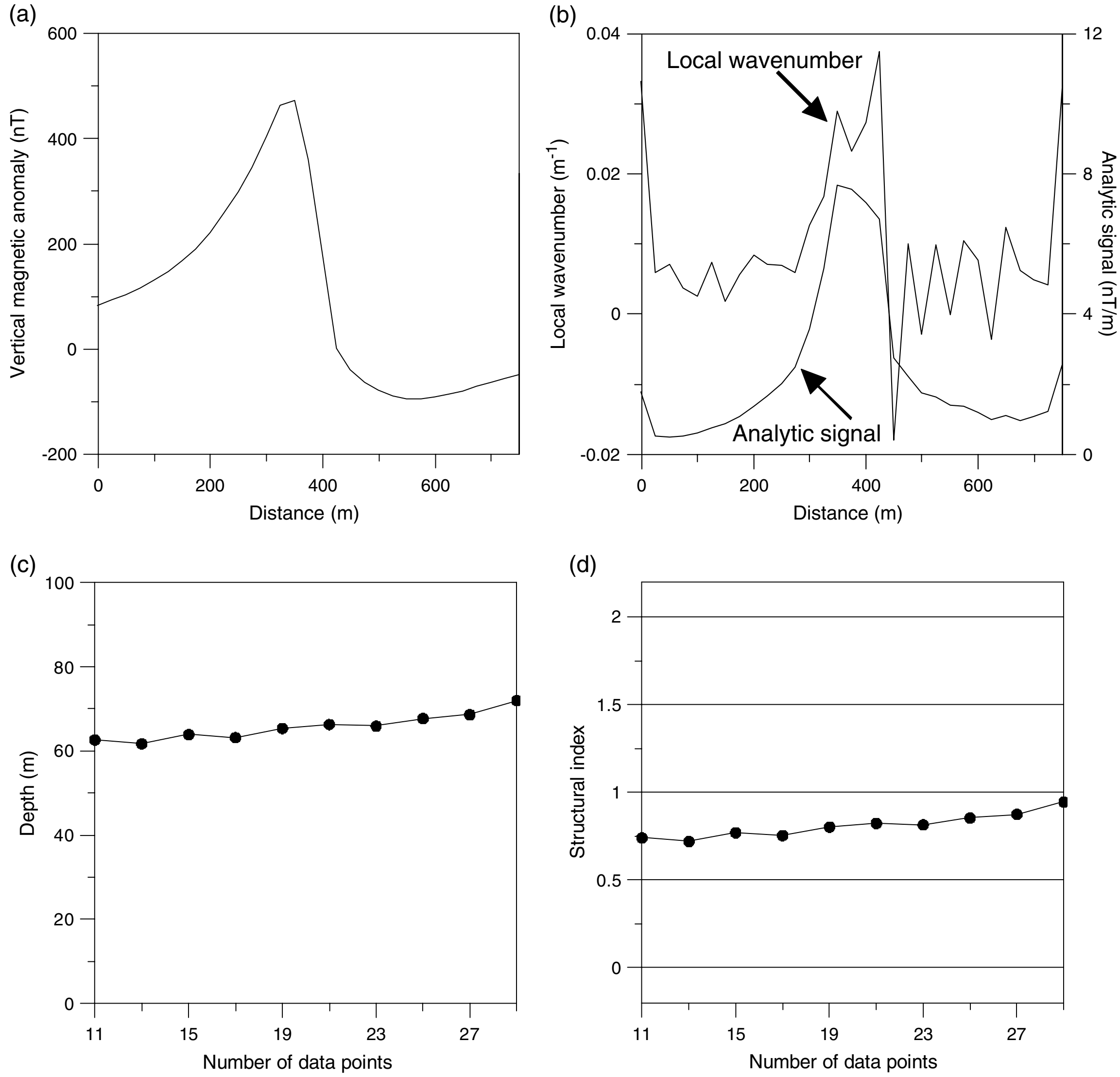

Figure 2 (a) Magnetic anomaly of the Pima copper deposit in Arizona (Gay 1963); (b) first-order local wavenumber and analytic signal; (c) estimate of the depth as a function of number of data points; (d) estimate of the structural index as a function of number of data points.

perpendicular to the geological strike. The source of the magnetic anomaly located near 700 m in Fig. 3(a) has been drilled, and the depth to the top of the magnetic body is 41 m below the surface. The local wavenumber profile (Fig. 3b) also displays high-frequency noise. As before, the horizontal position of the source was detected at a distance of 708 m using the analytic signal (Fig. 3b). The local wavenumber data were normalized and inversion was undertaken using different data windows ranging in size from 21 to 41 data points, in steps of 2 data points. Inversion results for the depth and structural index are illustrated in Figs 3(c) and 3(d), respectively. The estimate of the structural index is very close to 1, indicating that the source can be approximated by a thin dike model. This result agrees with Vallee *et al.* (2004), who estimated a structural index of 1.2 on the same anomaly. The estimated depth (about 45 m after subtraction of the sensor elevation of 76 m) agrees very well with a depth of 41 m from drilling information.

Example over anomalies of unknown sources

Figure 4(a) shows a profile of magnetic data collected for mineral exploration (Ontario Geological Survey 2002). The survey was in the Timmins area where the overburden is approximately 40 m deep (Irvine, Godbout and Witherly 2000),

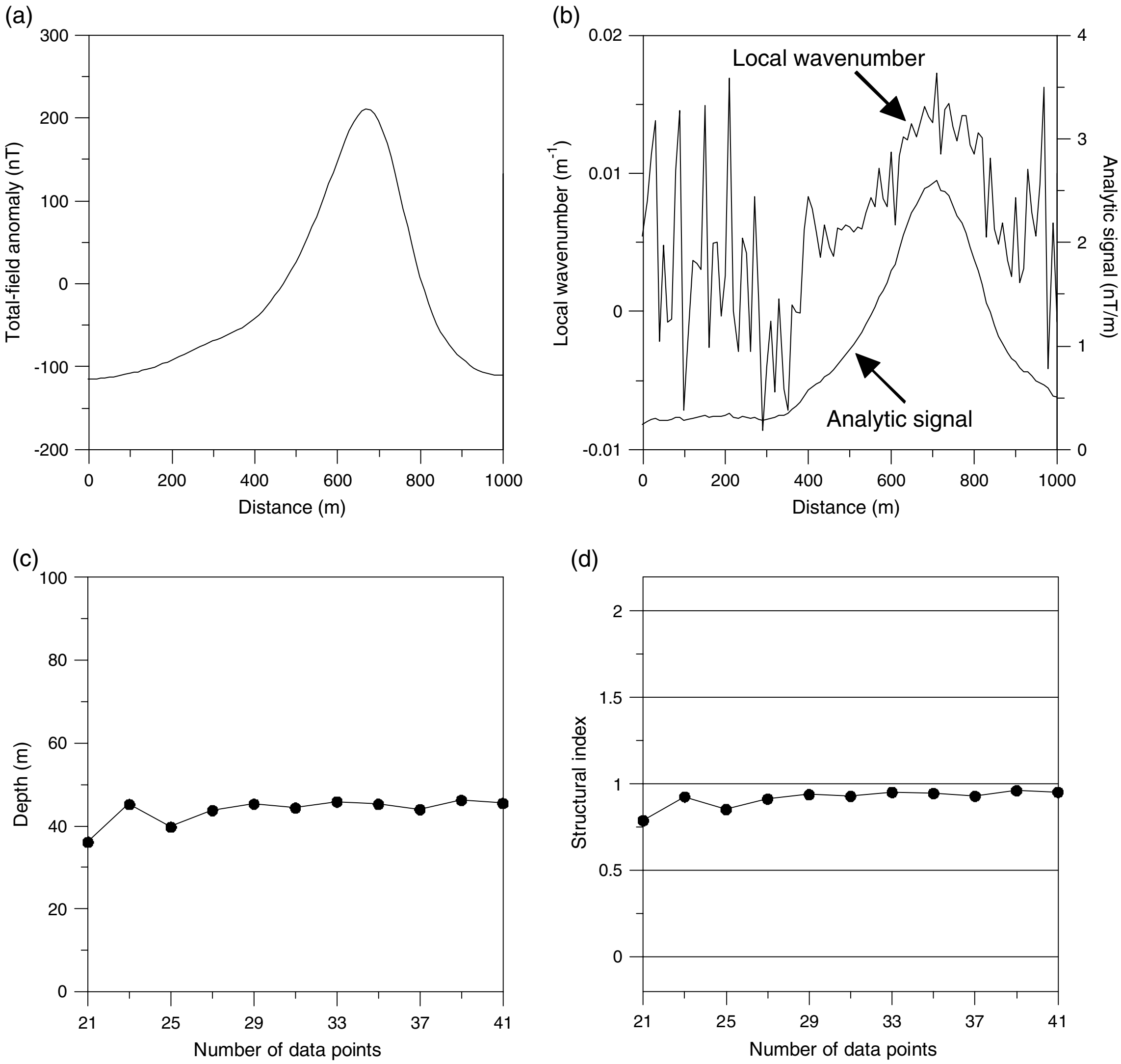

Figure 3 (a) Total-field magnetic anomaly for a magnetic body in the Matheson area, Canada; (b) first-order local wavenumber and analytic signal; (c) estimate of the depth as a function of number of data points; (d) estimate of the structural index as a function of number of data points.

but ranges from zero up to 100 m. The data were collected using a towed bird magnetometer nominally 70 m above the ground surface. Figure 4(b) shows the calculated first-order local wavenumber data. Noise levels are significant in this example. To reduce the effect of noise, we applied upward continuation for a distance of 20 m. We found that this distance is adequate to reduce the effect of the noise. Figure 4(c) shows the first-order local wavenumber data after applying the upward continuation filter. The positions of the wavenumber peaks were first identified. In the subsequent inversion for the depth and structural index, only data that are greater than 50% of each peak were used. Solutions were accepted if their indices lie between –0.2 and 2.2 (within the range of values expected for 2D magnetic bodies). Anomalies with indices that lie outside the acceptance range are most probably due to noise or very deep sources for which a 2D assumption would not be valid. The accepted solutions (labelled A, B, C, D and E) are listed in Table 1. Note that the tabulated depths are corrected for the sensor height and the upward continuation distance. The estimated depths all seem reasonable, as they are either close to the top of the magnetic basement, or below. Also, the estimated structural indices seem to be probable for this area as dikes are common, but other sources are possible.

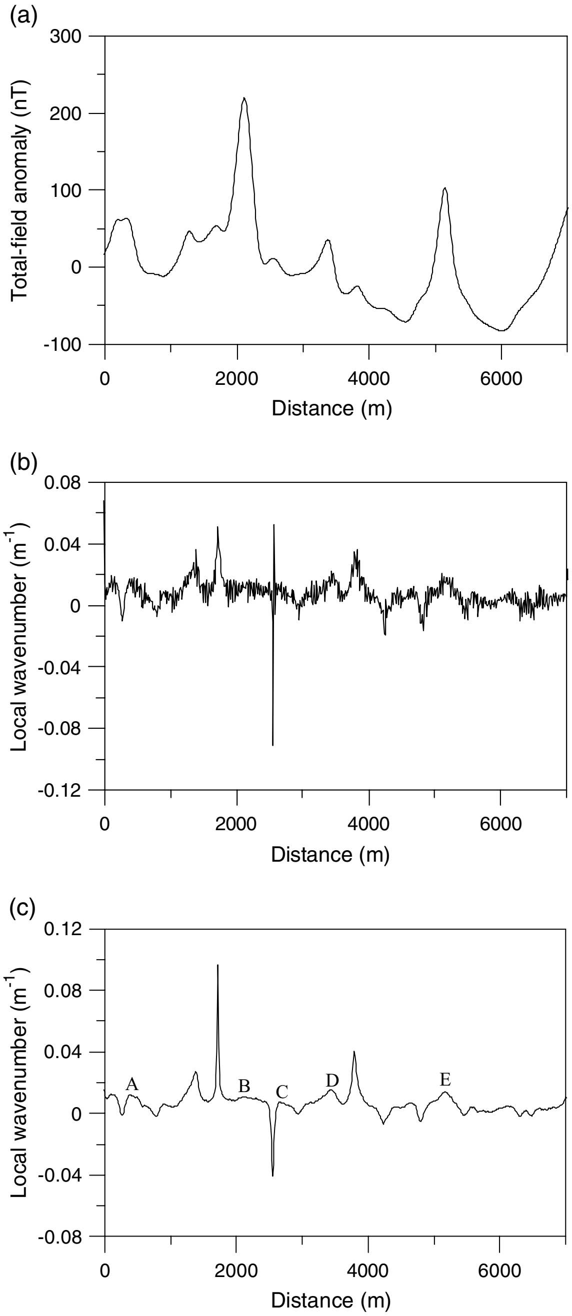

Figure 4 (a) Total-field magnetic profile from an airborne survey in the Timmins area; (b) first-order local wavenumber; (c) first-order local wavenumber after applying upward continuation for a distance of 20 m.

CONCLUSION

This paper presents a simple method for interpretation of magnetic data using normalization of the first-order local wavenumber of 2D magnetic anomalies. The normalized local wavenumber provides a generalized equation to estimate the depth without *a priori* information about the nature of the sources. Information about the nature of the sources is subsequently obtained using the source location parameters. The method was tested using theoretical and field examples, and good results were obtained. The disadvantage of the method is that it requires knowledge of the horizontal position of the source, which we inferred from the peak of the local wavenumber or the analytic signal. As a result, noise and/or inappropriate sampling of the data can make the selection of the horizontal position inaccurate, and this may lead to errors in the depth and structural index solutions.

Table 1 Results from the field example in Timmins area, Canada

Anomaly	Horizontal position (m)	Depth (m)	Structural index
A	420	84.8 ± 6	1.01 ± 0.12
B	2150	179.7 ± 10	1.86 ± 0.04
C	2710	80.0 ± 4	0.14 ± 0.07
D	3450	54.9 ± 1	1.19 ± 0.08
E	5170	61.5 ± 11	1.07 ± 0.05

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